SYNOPTIC: Field Equations Governing the Mechanical Behavior of Layered Circular Cylinders, F. H. Chou and J. D. Achenbach, Northwestern University, Evanston, Ill.; *AIAA Journal*, Vol. 8, No. 8, pp. 1445–1452.

Structural Mechanics and Materials; Structural Composite Materials

Theme

A system of governing equations is derived to describe the mechanical behavior of multilayered cylinders, consisting of alternating layers of a high-modulus reinforcing material and a low-modulus matrix material.

Content

In a multilayered hollow circular cylinder the number of distinct layers is large, and the thickness of an individual layer is thus small compared to the thickness of the cylinder. In investigating the static or dynamic response of a layered cylinder one can conceptually follow the approach of writing sets of governing equations for each layer and requiring their solutions to satisfy appropriate continuity conditions at the interfaces, as well as prescribed boundary conditions at the external surfaces of the cylinder. For a multilayered cylinder this approach is, however, not practical because of the rather serious analytical and computational difficulties that must be overcome if the number of layers is large. For practical purposes it is, moreover, generally not necessary to follow such a rigorous approach. Much interesting and important information on gross quantities such as the frequencies, as well as on local quantities such as the interface stresses, can be obtained by employing a gross continuum model for the layered medium.

In a continuum model the structuring of the medium is taken into account in an approximate manner in the governing field equations. Rather than analyze sets of field equations in each layer, we consider only one set of field equations for the layered cylinder, whatever the number of layers may be. The simplest continuum model is described by the "effective modulus theory." In this theory the mechanical behavior of the laminated medium is represented by an anisotropic continuum. For a layered cylinder with alternating layers of two homogeneous isotropic materials, shown in Fig. 1, the continuum model is homogeneous and transversely isotropic, with the axis of symmetry in the radial direction. Once the

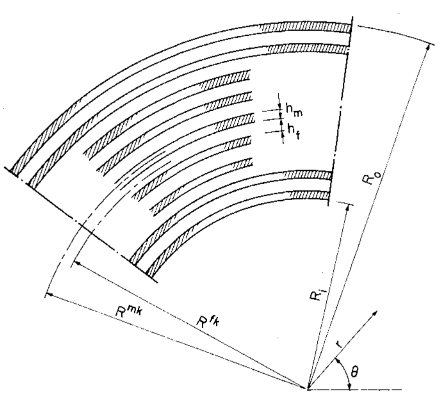

Fig. 1 Segment of a multilayered cylinder.

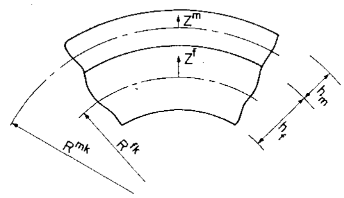

Fig. 2 Element of the *k*th pair of layers.

continuum model has been established, an analysis can proceed along a well-charted course. If this would be desirable, the field equations can then also be employed to derive a shell theory.

In this paper we go one step beyond the effective modulus approach in that the effect of local deformations is taken into account in the continuum model. The system of governing equations for the homogeneous continuum model of the laminated medium with curvature is derived in two stages. The first stage of the derivation involves certain assumptions and operations within the system of discrete, curved layers. In particular, it is assumed that the displacements in the individual layers can be described by two-term expansions in the local coordinates normal to the mid-surfaces of the layers (see Fig. 2). The kinematic variables that are introduced in the expansions are thus defined at the midsurfaces of the layers only. Also in the system of discrete layers, balance equations of linear momentum and moment of momentum for the individual layers are obtained by integrating the local balance equations across the thicknesses of the layers. These integrations lead to the definitions of average stresses and couple-stresses which are again defined in discrete surfaces only. In the next stage of the derivation a transition is made from the system of discrete layers to the homogeneous continuum model. The transition is accomplished by defining fields for the kinematical and dynamical variables that are continuous in the radial coordinate. In prescribed discrete circular cylindrical surfaces the field variables assume the same values as the variables that were defined in the discrete system of layers.

Within the continuum model the state of deformation is described by the gross displacement and by local deformations. These fields are related by conditions representing continuity of the local displacements at the interfaces. We also construct potential and kinetic energy densities for the continuum model. In conjunction with Hamilton's principle the energy densities are employed to derive displacement equations of motion, where the continuity conditions enter by the use of Lagrangian multipliers. The theory is completed by defining constitutive equations and boundary conditions. The resulting system of field equations is similar in form to that of a theory of elasticity with microstructure. It is shown that it contains the effective modulus theory as a special limit case.